

Robust forecasting of electricity prices with nonlinear models and exogenous regressors

Luigi Grossi, Fany Nan

Abstract It is well known that GM estimators for linear models are consistent and lead to a small loss of efficiency with respect to LS estimator. When they are extended to threshold models, which are piecewise linear models, the consistency of GM estimators is guaranteed only under certain choices of the objective function. In this paper we suggest the use of robust SETAR (Self Exciting Threshold AutoRegressive) processes to model and forecast electricity prices observed on deregulated markets. The main advantages of estimating robust SETAR models is the possibility to capture two very well-known stylized facts of electricity prices: nonlinearity produced by changes of regimes and the presence of sudden spikes due to inelasticity of demand.

Key words: Electricity prices, extreme values, GM estimator, nonlinear models

1 Introduction

A very well known stylized fact of electricity prices is the presence of isolated jumps as a consequence of sudden grid congestions which reflects immediately on prices because of lack of flexibility of the supply and demand curves. This feature must be considered very carefully and robust techniques must be applied to avoid that few jumps could dramatically affect parameter estimates. Although many papers have applied quite sophisticated time series models to prices and demand time series of electricity and gas only few have considered the strong influence of jumps on estimates and the need to move to robust estimators (Janczura et al.; 2013). Among robust techniques for electricity prices, robust SETAR models have never been estimated. The reasons could be resumed by two main points:

- 1) properties of robust SETAR estimators have not been completely studied and there isn't a clear accordance on the best estimator, at least with reference to the best weighting function;

- 2) robust estimators are not implemented within the most popular statistical software platforms such as matlab and R.

Grossi and Nan (2013) have addressed the two points through a massive Monte Carlo experiment which compares the performances of classical SETAR estimator

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and robust estimators using different weighting functions. All the estimators (classical and robust) have been implemented in R language resulting in a set of functions which hopefully will become a library soon.

In this paper, using the results contained in Grossi and Nan (2013), classical and robust estimators are applied to estimate parameters of SETAR models on Italian electricity price data (*PUN, prezzo unico nazionale*). The model is enriched by the introduction of exogenous regressors which should improve the forecasting performances. Crucial variables in predicting electricity prices are dummies for the intra-day seasonality and demanded volumes (Gianfreda and Grossi; 2012). Comparisons will be made among different robust estimators and between pure SETAR models and nonlinear specifications with exogenous regressors.

The paper is organized as follows. In the next section, the general SETAR model is introduced and the main weighting functions are discussed to move to robust estimators. Section three contains summary and comments of the forecasting results. Conclusion and final remarks are reported in section four.

2 Robust SETAR models with exogenous regressors

Given a time series y_t , a general two-regime Self-Exciting Threshold AutoRegressive model SETAR(p, d) with exogenous regressors is specified as

$$y_t = (\mathbf{x}_t \beta_1 + \mathbf{z}_t \lambda_1) I(y_{t-d} \leq \gamma) + (\mathbf{x}_t \beta_2 + \mathbf{z}_t \lambda_2) I(y_{t-d} > \gamma) + \varepsilon_t \quad (1)$$

for $t = 1, \dots, N$, where $I(\cdot)$ is an indicator function, y_{t-d} is the threshold variable with $d \geq 1$ and γ is the threshold value. The relation between y_{t-d} and γ states if y_t is observed in regime 1 or 2. β_j is the vector of auto-regressive parameters for regime $j = 1, 2$ and $\mathbf{x}_t$ is the t -th row of the $(N \times p)$ matrix $\mathbf{X}$ comprising p lagged variables of y_t (and possibly a constant). λ_j is the vector corresponding to exogenous regressors contained in the $(N \times r)$ matrix $\mathbf{Z}$ whose t -th row is $\mathbf{z}_t$. Errors ε_t are assumed to follow an iid($0, \sigma_\varepsilon$) distribution.

In general the value of the threshold γ is unknown, so that the parameters to estimate become $\theta = (\beta'_1, \beta'_2, \lambda'_1, \lambda'_2)$, γ and σ_ε . Parameters can be estimated by sequential conditional least squares: for a fixed threshold γ the model is linear, θ can be estimated by OLS and $\hat{\sigma}_\varepsilon = \sqrt{\sum_{t=1}^N r_t^2 / N}$, with $r_t = y_t - \mathbf{x}_t^* \hat{\theta}$, where $\mathbf{x}_t^* = (\mathbf{x}_t, \mathbf{z}_t)$. The least square estimate of γ is obtained by minimizing the residual sum of squares $\gamma = \arg \min_{\gamma \in \Gamma} \sum_{t=1}^N r_t^2$ over a set Γ of allowable threshold values so that each regime contains at least a given fraction (ranging from 0.05 to 0.3) of all observations.

In the case of robust two-regime SETAR model, for a fixed threshold γ the GM estimate of the autoregressive parameters can be obtained by applying the iterative weighted least squares:

$$\hat{\theta}_j^{(n+1)} = \left(\mathbf{X}_j^{*'} \mathbf{W}^{(n)} \mathbf{X}_j^* \right)^{-1} \mathbf{X}_j^{*'} \mathbf{W}^{(n)} \mathbf{y}_j \quad (2)$$

where $\hat{\theta}_j^{(n+1)}$ is the GM estimate for the parameter vector in regime $j = 1, 2$ after the n -th iteration from an initial estimate $\hat{\theta}_j^{(0)}$, and $\mathbf{W}^{(n)}$ is a weight diagonal matrix, whose elements depend on a weighting function $w(\hat{\theta}_j^{(n)}, \hat{\sigma}_{\varepsilon,j}^{(n)})$ bounded between 0 and 1. The threshold γ can be estimate by minimizing an objective function (see 2.1) over the set Γ of allowable threshold values.

2.1 The weighting methods

The first method is described in Chan and Cheung (1994). Weights are calculated as

$$w(\hat{\theta}_j, \hat{\sigma}_{\varepsilon,j}) = \psi\left(\frac{y_t - m_{y,j}}{C_y \hat{\sigma}_{y,j}}\right) \psi\left(\frac{y_t - \mathbf{x}_t^* \hat{\theta}_j}{C_\varepsilon \hat{\sigma}_{\varepsilon,j}}\right)$$

where ψ is the Tukey bisquare weighting function and $m_{y,j}$ is a robust estimate of the location parameter (sample median) in the j -th regime. $\hat{\sigma}_{y,j}$ and $\hat{\sigma}_{\varepsilon,j}$ are robust estimates of the scale parameters σ_y and σ_ε respectively, obtained by the median absolute deviation multiplied by 1.483. C_y and C_ε are tuning constants fixed at 6.0 and 3.9 respectively.

The objective function to minimize for the search of the threshold depends on Tukey bisquare weights. We use the same function described in Chan and Cheung (1994).

For the second method, we follow Franses and van Dijk (2000). The GM weights are presented in Schweppe's form $w(\hat{\theta}_j, \hat{\sigma}_{\varepsilon,j}) = \psi(r_t)/r_t$ with standardized residuals $r_t = (y_t - \mathbf{x}_t^* \hat{\theta}_j)/(\hat{\sigma}_{\varepsilon,j} w(\mathbf{x}_t^*))$ and $w(\mathbf{x}_t^*) = \psi(d(\mathbf{x}_t^*)^\alpha)/d(\mathbf{x}_t^*)^\alpha$. $d(\mathbf{x}_t^*) = |\mathbf{x}_t^* - m_{y,j}|/\hat{\sigma}_{y,j}$ is the Mahalanobis distance and α is a constant usually set equal to 2 to obtain robustness of standard errors. The chosen weighting function is the polynomial ψ function. The threshold γ is estimated by minimizing the objective function $\sum_{t=1}^N w(\hat{\theta}, \hat{\sigma}_\varepsilon)(y_t - \mathbf{x}_t^* \hat{\theta})^2$ over the set Γ of allowable threshold values.

The third method is based on the same methodologies of the second but with ψ being the Huber weighting function.

3 Application: Italian electricity price

In this section, we apply LS and three robust weighting functions, presented in previous section, to estimate parameters of SETAR models on Italian electricity price data (*PUN, prezzo unico nazionale*). Moreover, a comparison of prediction accuracy among the methods is conducted.

The time series of prices used in the present work covers the period from January 1st, 2010 to December 31th, 2012 (26,304 data points, for $N = 1,096$ days): year 2012 has been left for out-of-sample forecasting. The data have an hourly frequency, therefore each day consists of 24 load periods with 00:00–01:00am defined as period

1. Spot price is denoted as P_{tj} , where t specifies the day and j the load period ($t = 1, 2, \dots, N$; $j = 1, 2, \dots, 24$).

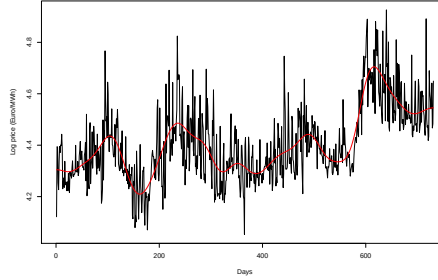
In this study, following a widespread practice in literature, each hourly time series is modeled separately.

Differences in load periods can cause significant variations in price time series. A first inspection, based on graphs, spectra and ACFs (Figures are not reported for lack of space but can be obtained from the authors) for different hours, shows that the series have long-run behaviour and annual dynamics, which change according to the load period. A common characteristic of price time series is the weekly periodic component (of period 7), suggested by the spectra that show three peaks at the frequencies $1/7$, $2/7$ and $3/7$, and a very persistent autocorrelation function.

We assume that the dynamics of log prices can be represented by a nonstationary level component L_{tj} , accounting for level changes and/or long-term behaviour, and a residual stationary component p_{tj} , formally, $\log P_{tj} = L_{tj} + p_{tj}$.

To estimate L_{tj} we used the wavelets approach (Percival and Walden; 2000). Wavelets have been used in many studies, including Janczura and Weron (2010) and Trück et al. (2007). We considered the Daubechies least asymmetric wavelet family, LA(8), and the coefficients were estimated *via* the maximal overlap discrete wavelet transform (MODWT) method (for details, see Percival and Walden; 2000). Figure 1 shows $\log P_{tj}$ for hour 21, with the estimated nonstationary level component superimposed.

Fig. 1 $\log P_{tj}$ for hour 21 (9pm), with the estimated nonstationary level component superimposed.



We applied two different approaches. First after removing the long-term component, we estimated on the stationary time series p_{tj} the SETAR(p, d) model with exogenous regressors, as reported in eq. (1). Secondly, we use the estimated long-term component as a regressor in a model where $\log P_{tj}$ is the dependent variable. According to the empirical ACFs, a SETAR(7,1) model has been estimated over all the price series to highlight differences in the estimation given by different dynamics characterizing each load period. Matrix $\mathbf{Z}$ of exogenous regressors could contain day-of-the-week dummies, D_k , with $k = 1, \dots, 6$ and the day-ahead predicted demand of electricity made available by GME (Gestore Mercato Elettrico). Next step of the analysis will be to compare the forecasting performances of the robust methods with the forecasting performance of the LS estimator.

For comparing our robust/non robust SETAR models, we reproduced 366 one day-ahead forecasts $\hat{p}_{t+1}$ for each model estimated on a rolling window of 2 years. Comparisons are based on the predictions of the original spot prices that are given by $\hat{P}_{t+1} = \exp(\hat{L}_{t+1} + \hat{p}_{t+1})$, where $\hat{L}_{t+1}$ and $\hat{p}_{t+1}$ are predictions of the components, which are based only on the information available in t . In particular, we set $\hat{L}_{t+1} = \hat{L}_t$, that is, we used the estimated value in t as a forecast for $t + 1$. Besides its simplicity, the motivation to use this equation comes from the fact that the long-term component, by definition, should be basically the same for two contiguous days. Forecasts have been compared in terms of MSE (Mean Square Error) and MAE (Mean Absolute Error), and of the Diebold and Mariano test, that are based on the forecasting errors $e_{tj} = P_{tj} - \hat{P}_{tj}$, ($t = 1, 2, \dots, M$; $j = 1, 2, \dots, 24$) for each method. We used the one-tailed Diebold and Mariano test (DM), whose null hypothesis is that the prediction accuracy of procedure (say) A is equal or lower than that of procedure B. The test has been performed both with MSE and MAE.

Table 1 Hour 21, model SETAR(7,1) without exogenous regressors. MSE and MAE of forecasts obtained from the four models on the four estimation periods and on the whole. In parenthesis the models whose forecasts are statistically worse than predictions of the model in the column (1-tailed Diebold and Mariano test at 5% significance level, MSE and MAE loss functions.)

Period	MSE				MAE			
	LS	POL	HUB	TUK	LS	POL	HUB	TUK
Jan-Mar	200.7	214.58	206.83	195.80	10.06	9.93	9.94	9.80
Apr-Jul	159.12	148.20	148.71	153.63	9.61	9.49	9.41	9.55
Aug-Sept	688.24	696.87	686.51	675.47	14.57	13.49	13.81	13.66
Oct-Dec	67.29	65.18	64.35	65.26	6.04	(LS,HUB)	(LS)	(LS)
						5.71	5.72	5.72
Year	279.38	281.75	277.14	273.07	10.07	(LS)	(LS)	(LS)
						9.65	9.72	9.68
						(LS)	(LS)	(LS)

Table 2 Hour 21. Ratios of MSE and MAE of forecasts obtained with models Exo1 and Exo2 to MSE and MAE of forecasts obtained with model SETAR(7,1) without exogenous regressors (four estimation periods and whole year).

Period	Exo1				Exo2			
	MSE		MAE		MSE		MAE	
	LS	POL	LS	POL	LS	POL	LS	POL
Jan-Mar	1.049	0.975	0.975	0.968	0.930	0.910	0.910	0.954
Apr-Jun	1.002	1.051	1.029	1.007	1.038	1.071	0.997	1.031
Jul-Sep	1.041	0.992	0.947	0.956	1.069	0.981	1.062	1.048
Oct-Dec	0.914	0.885	0.960	0.977	1.024	1.012	0.973	1.015
Year	1.029	0.990	0.976	0.974	1.037	0.981	0.995	1.015

Table 1 shows MSE and MAE values on the whole year 2012 and on the four seasons. In parenthesis we reported the models whose forecasts are statistically worse than predictions of the model in the column considering the 1-tailed Diebold and Mariano test at 5% significance level and MSE and MAE loss functions.

Table 2 shows a forecasting assessment of SETAR models with different regressor sets: in case “Exo1” the dependent variable $\hat{p}_{t,j} = \log P_{t,j} - \hat{L}_{t,j}$ is regressed on the logarithm of the predicted demand adjusted from its level component to obtain a stationary series. The level component has been estimated with the same approach used for $\log P_{t,j}$. In case “Exo2” variable $\log P_{t,j}$ is regressed on the logarithm of the predicted demand and $\hat{L}_{t,j}$. Values reported in the table are ratios of MSE (MAE) of the SETAR with exogenous regressors and pure SETAR. As can be seen, robust models with regressors show better forecasting performance than the pure SETAR model, but further analysis is needed to achieve stable and more general conclusions.

4 Conclusions

In this paper the the forecasting performances of robust estimators applied to electricity prices have been studied. Dummy variables have been included in the model in order to account for the presence of seasonality in the data and day-ahead predicted demand has been considered to test its relevance in predicting day-ahead prices. Summarizing the main results we could say that robust estimators overperform the classical Least Squares estimator, with a slight preference of Polynomial on Huber’s weights. The use of regressors seems to increase the forecasting power of models, but deeper analysis is needed to argue the significance of the improvement of predictions.

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